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This document contains instructions for using the Hertzian Biphasic Theory (HBT) Template in Microsoft Excel®. The template is designed to fit a single creep indentation curve to the HBT method [1], appearing in the journal of tribology. The HBT model was originally published in 2014 [2], appearing in the journal of biomechanics. The method and similarly the template are based on the experimental configuration of a rigid impermeable sphere contacting a biphasic layer bonded to a rigid substrate.

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I. Overview:

The following is a brief description of each tab in the file.

HBT strain dependent k

The HBT strain dependent k tab is designed to fit a single creep curve to the Hertzian Biphasic Theory (HBT) using strain dependent permeability [1]. Inputs include probe radius, tissue thickness, time, deformation, normal force, and initial guesses. The template Outputs the compressive modulus (Ey-), tensile modulus (Ey+), coefficient of determination (R^2), and the strain dependent permeability parameters (M and k0).

HBT constant k

The HBT constant k tab is designed to fit a single creep curve to the Hertzian Biphasic Theory (HBT) using constant permeability [1]. Inputs include probe radius, tissue thickness, time, deformation, normal force, and an initial guess. The template Outputs the compressive modulus (Ey-), tensile modulus (Ey+), permeability (k), and coefficient of determination (R^2).

Oyen (substrate correction)

The Oyen (substrate correction) tab is designed to fit a single creep curve to the Oyen method [3]. The Oyen method is adjusted to include substrate effects [4]. Inputs include probe radius, creep load, tissue thickness, time, deformation, and initial guesses. The template Outputs the permeability (k), quasi Poisson's ratio, Young's modulus (Ey), shear modulus (G), effective contact modulus (Ec'), contact modulus (Ec), and coefficient of determination (R^2).

II. Solver Function:

This section describes how to install the Solver function in Microsoft Excel®. This section is specifically catered to Microsoft Excel 2010 but can be downloaded for both previous and later versions of this software. The Solver uses numerical iteration on one or more variables to achieve a target value.

Install Solver

Go to **File > Options > Add-Ins > Manage: Excel Add-ins > Go >** check the box for **Solver Add-in > OK**.

Solver Location

The Solver function can now be found in the **Data** tab, on the far right hand side.

Running the Solver

To use the Solver you must **Set Objective** cell and choose to either **Maximize**, **Minimize**, or set a target **Value Of**. Constraints can then be added based on your specific material; while we do not discuss them here they can prevent the solver from failing. Check the box **Make Unconstrained Variables Non-Negative** unless one or more of the cells can be negative. Select **GRG Nonlinear** (default) and then **Solve**.

Solver Output

The user now has options to keep the solution, revert to the original values, or return to the Solver to adjust parameters. Assuming a successful iteration select **OK**. In the template any tab that has raw data as an input computes the coefficient of determination R^2 which is a measure of how well the model being used fits the data. The closer the R^2 value is to 1 the better the agreement.

III. Fitting HBT strain dependent permeability:

This section describes the process for fitting a single creep (force controlled) curve to HBT. The **HBT strain dependent k** tab is used in this section.

Note that creep relaxation, a hybrid of creep and stress relaxation, can also be used in this section. For more information on creep relaxation please see ref [1].

Step 1

The HBT method defines the elastic constants, tensile modulus (Ey+) and compressive/contact modulus (Ey-), under the assumption of instantaneous loading and full equilibrium, respectively. For this reason the initial loading sequence must be performed in a nearly instantaneous manner. In addition the sample should be given plenty of time to approach full equilibrium, see ref [1] for equilibrium criteria.

Step 2

The model **INPUTS** can be put into the template. The template is color coded: 'tan' colored cells are for user **INPUTS**.

INPUT: probe radius (mm), tissue thickness (mm), time (s), deformation (mm), and normal force (N).

Note that only one creep curve can be fit at a time and the user must delete all of the old data (time, deformation, and normal force) prior to starting a new fit. An example data set is provided in the template.

Step 3

The Solver function can now be run to optimize HBT. Open the Solver. We want to maximize the coefficient of determination R^2 by changing the initial guesses M, k0, Ey-, and Ey+.

`Set Objective:` **$E$7**

`To:` **Max**

`By Changing Variable Cells:` **$B$7:$B$8, $E$5:$E$6**

`Subject to the constraints:` **$B$8 >= 0**
$E$11 = $B$11
$E$5 <= $H$5

`Make Unconstrained Variables Non-Negative:` **Yes**

Select a Solving Method: **GRG Nonlinear**

Step 4

Assuming a successful iteration the template is complete. The **OUTPUTS**, light blue, are **strain dependent permeability constants M and k0 (mm^4/Ns), Ey- (MPa), Ey+ (MPa), R^2, and a plot with the deformation data and model fit.**

Note that the strain dependent model has the form: k = k0*e^(-M*strain). This model comes from ref [5].

Troubleshooting

There is one main error that may occur in this tab.

1) Solver cannot converge
    a. Check your objective and variable cells.
    b. Directly manipulate M, k0, Ey- and Ey+ to bring the model fit closer to the data as it may be too far out of range. Initial guesses for these parameters can be found in cells H5:H7.
    c. Make sure there is only one creep indentation present in the cells.

IV. Fitting HBT constant permeability:

This section describes the process for fitting a single creep (force controlled) curve to HBT. The **HBT constant k** tab is used in this section.

Note that creep relaxation, a hybrid of creep and stress relaxation, can also be used in this section. For more information on creep relaxation please see ref [1].

Step 1

The HBT method defines the elastic constants, tensile modulus (Ey+) and compressive/contact modulus (Ey-), under the assumption of instantaneous loading and full equilibrium, respectively. For this reason the initial loading sequence must be performed in a nearly instantaneous manner. In addition the sample should be given plenty of time to approach full equilibrium, see ref [1] for equilibrium criteria.

Step 2

The model **INPUTS** can be put into the template. The template is color coded: 'tan' colored cells are for user **INPUTS**.

INPUT: probe radius (mm), tissue thickness (mm), time (s), deformation (mm), and normal force (N).

Note that only one creep curve can be fit at a time and the user must delete all of the old data (time, deformation, and normal force) prior to starting a new fit. An example data set is provided in the template.

Step 3

The Solver function can now be run to optimize HBT. Open the Solver. We want to maximize the coefficient of determination R^2 by changing the initial guesses k, Ey-, and Ey+.

`Set Objective:` **$E$8**

`To:` **Max**

`By Changing Variable Cells:` **$E$5:$E$7**

`Subject to the constraints:` **$E$11 = $B$11**
$E$5 <= $H$5

`Make Unconstrained Variables Non-Negative:` **Yes**

`Select a Solving Method:` **GRG Nonlinear**

Step 4

Assuming a successful iteration the template is complete. The **OUTPUTS**, light blue, are **Ey- (MPa), Ey+ (MPa), k (mm^4/Ns), R^2 and a plot with the deformation data and model fit.**

Troubleshooting

There is one main error that may occur in this tab.

1) Solver cannot converge
    a. Check your objective and variable cells.
    b. Directly manipulate Ey-, Ey+ and k to bring the model fit closer to the data as it may be too far out of range. Initial guesses for these parameters can be found in cells H5:H7.
    c. Make sure there is only one indentation present in the cells

V. Fitting Oyen method with substrate correction:

This section describes the process for fitting a single creep (force controlled) curve to Oyen's model. The **Oyen (substrate correction)** tab is used in this section.

Step 1

The Oyen method assumes instantaneous loading of the tissue. For this reason the initial loading sequence must be performed in a nearly instantaneous manner. The Oyen method, however, does not require full equilibrium as it predicts the equilibrium deformation from an analytical curve fit.

Step 2

The model **INPUTS** can be put into the template. The template is color coded: 'tan' colored cells are for user **INPUTS**.

INPUT: probe radius (mm), creep load (N), tissue thickness (mm), time (s), and deformation (mm).

Note that only one creep curve can be fit at a time and the user must delete all of the old data (time and deformation) prior to starting a new fit. An example data set is provided in the template.

Step 3

The Solver function can now be run to optimize the Oyen method. Open the Solver. We want to maximize the coefficient of determination R^2 by changing the initial guesses for predicted maximum displacement and k.

`Set Objective:` **$E$10**

`To:` **Max**

`By Changing Variable Cells:` **$B$8:$B$9**

`Make Unconstrained Variables Non-Negative:` **Yes**

`Select a Solving Method:` **GRG Nonlinear**

Step 4

Assuming a successful iteration the template is complete. The **OUTPUTS**, light blue, are **k (mm^4/Ns), quasi Poisson's ratio, Young's modulus Ey (MPa), shear modulus G (MPa), effective contact modulus Ec' (MPa), contact modulus Ec (MPa), R^2, and two plots: 1) recreated deformation data and model fit and 2) dimensionless data and model fit.**

Troubleshooting

There is one main error that may occur in this tab.

1) Solver cannot converge
    a. Check your objective and variable cells.
    b. Directly manipulate the predicted maximum displacement and k values to bring the model fit closer to the data as it may be too far out of range.
    c. Make sure there is only one indentation present in the cells

Notes

1) The Oyen model generally uses a lot of computing power and can run for 10-20 minutes.
2) In addition, the quasi Poisson's ratio can go negative and give a negative Young's modulus for reasons discussed in ref [1].

VI. References

[1] Moore, A. C., DeLucca, J. F., Burris, D. L., and Elliott, D. M., 2015, "Quantifying cartilage contact modulus, tensile modulus, and permeability with Hertzian biphasic creep," Journal of Tribology.
[2] Moore, A. C., and Burris, D. L., 2014, "An analytical model to predict interstitial lubrication of cartilage in migrating contact areas," Journal of Biomechanics, 47(1), pp. 148-153.
[3] Oyen, M. L., 2008, "Poroelastic nanoindentation responses of hydrated bone," Journal of Materials Research, 23(5), pp. 1307-1314.
[4] Stevanovic, M., Yovanovich, M. M., and Culham, J. R., 2001, "Modeling Contact between Rigid Sphere and Elastic Layer Bonded to Rigid Substrate," IEEE TRANSACTIONS ON COMPONENTS AND PACKAGING TECHNOLOGIES, 24, pp. 207-212.

[5] Lai, W. M., and Mow, V. C., 1980, "Drag-Induced Compression of Articular-Cartilage during a Permeation Experiment," Biorheology, 17(1-2), pp. 111-123.