



**This document contains instructions for using the Biphasic Model Template in Microsoft Excel®. The template is designed to fit rate controlled indentations to the spherical indentation model described in [1], appearing in the scientific journal tribology international. The model and similarly the template are based on the experimental configuration of a rigid impermeable sphere contacting a biphasic layer bonded to a rigid substrate.**

AC Moore, DL Burris

December 10, 2014

<b>I</b>	Overview	1
<b>II</b>	Solver Function	1
<b>III</b>	Fitting Hertz	1
<b>IV</b>	Biphasic Model Fit	3
<b>V</b>	References	4

## I. **Overview:**

The following is a brief description of each tab in the file.

### **Fitting Hertz**

The Fitting Hertz tab is designed to fit a single rate controlled indentation curve to Hertz's Theory. INPUTS include probe radius, tissue thickness, normal force, deformation and initial guesses. The template OUTPUTS the effective contact modulus.

### **Biphasic Model Fit**

The Biphasic Model Fit tab is designed to fit multiple rate controlled indents to the biphasic model explained in the article [1]. INPUTS include probe radius, equilibrium contact modulus, indentation rate, effective contact moduli and initial guesses. The template OUTPUTS the permeability and tensile modulus.

## II. **Solver Function:**

This section describes how to install the Solver function in Microsoft Excel®. This section is specifically catered to Microsoft Excel 2010 but can be downloaded for both previous and later versions of this software. The Solver uses numerical iterations on one or more variables to achieve a target value.

### **Install Solver**

Go to **File > Options > Add-Ins > Manage: Excel Add-ins > Go** > check the box for **Solver Add-in > OK**.

### **Solver Location**

The Solver function can now be found in the **Data** tab, on the far right hand side.

### **Running the Solver**

To use the Solver you must **Set Objective** cell and choose to either **Maximize**, **Minimize**, or set a target **Value Of**. Constraints can then be added based on your specific material; while we do not discuss them here they can prevent the solver from failing. Generally it is not necessary to check the box **Make Unconstrained Variables Non-Negative**. Select **GRG Nonlinear** (default) and then **Solve**.

### **Solver Output**

The user now has options to keep the solution, revert to the original values, or return to the Solver to adjust parameters. Assuming a successful iteration select **OK**. In the template any tab that has raw data as an input computes the coefficient of determination  $R^2$  which is a measure of how well the model being used fits the data. The closer the  $R^2$  value is to 1 the better the agreement.

## III. **Fitting Hertz:**

This section describes the process for fitting a single force-displacement (indentation) curve to Hertz's Theory. The **Fitting Hertz** tab is used in this section.

### **Step 1**

Articular cartilage presents several measurement difficulties when performing indentation. First cartilage has a poorly defined surface due to a highly compliant and irregular surface layer. Second cartilage weeps fluid during contact; this loss of fluid creates suction forces that produce highly non-Hertzian behavior during the retraction of the probe [1]. Third cartilage acts like an elastic layer bonded to a rigid substrate. For these reasons we make three data processing decisions. 1) When indenting we do not define the zero surface, instead we place the probe  $\sim 3\mu\text{m}$  above the surface and then indent and retract. 2) Only the forward part of indentation is used; the retraction is discarded. 3) We correct the measured force data based on the layer thickness and contact radius as described by Stevanovic et al. [2].

The only pre-processing step the user must do is input the forward part of indentation. The surface offset and layer correction factor is automatically calculated.

### Step 2

With the pre-processed data the **INPUTS** can be put into the template. The template is color coded; 'tan' colored cells are for user **INPUTS**.

**INPUT: probe radius (mm), tissue thickness (mm), normal force (N), deformation (mm) and initial guesses for contact modulus and surface offset (mm)**

Note that only one indent can be fit at a time.

### Step 3

The Solver function can now be run to optimize Hertz's Theory. Open the Solver. We want to minimize the error by changing the contact modulus and zero surface offset (tan colored cells).

Set Objective: **\$J\$10**

To: **Min**

By Changing Variable Cells: **\$H\$10:\$I\$10**

Make Unconstrained Variables Non-Negative: **No**

Select a Solving Method: **GRG Nonlinear**

### Step 4

Assuming a successful iteration the template is complete. The **OUTPUTS**, light blue, are the **coefficient of determination  $R^2$ , surface offset (mm), contact modulus (MPa), and a plot with the original data, substrate corrected data and model fit.**

### Troubleshooting

There is one main error that may occur in this tab.

- 1) Solver cannot converge
  - a. Check your objective and variable cells.
  - b. Change your initial guesses as they may be out of range.
  - c. Try setting the initial guess for contact modulus to a very low value and the initial guess for zero surface offset to 0mm and let the fitting program figure it out.

- d. Make sure there is only one indentation present in the cells
- e. Try checking the box to **Make Unconstrained Variables Non-Negative**.
- f. Try fitting only a fraction of the approach, i.e. upper 80%. If this works see how much of the original data can be kept by slowly increasing the percentage.
- g. Data is extremely non-Hertzian. The fitting program is pretty robust and should be able to fit most indentation data, but do not expect all materials to work.

#### IV. Biphasic Model Fit:

This section describes the process for fitting a series of rate controlled indents to the biphasic model as described by the article [1]. The **Biphasic Model Fit** tab is used in this section. To run this template it is assumed that a series of rate controlled indents were performed on a single sample.

##### Step 1

Each of the rate controlled indents should have been fit to Hertz's Theory using the **Fitting Hertz** tab to determine the effective contact modulus.

##### Step 2

The **INPUTS** can be placed into the template. The template is color coded, 'tan' colored cells are for user **INPUTS**.

**INPUT: probe radius (mm), equilibrium contact modulus (MPa), indentation rate (mm/s), effective contact modulus (MPa), and initial guesses for tensile modulus (MPa) and permeability (mm<sup>4</sup>/Ns).**

Do not place the equilibrium modulus in the contact modulus cells.

Note that the equilibrium contact modulus can be found by indenting the probe in to the sample and waiting until equilibrium is reached, i.e. the change in normal force and/or the change in deformation as a function of time = 0. Since the equilibrium contact modulus is a single data point we can use Hertz's equation coupled with the layer correction factor to determine the equilibrium modulus directly using the equilibrium calculator on this page.

To use the equilibrium calculator the user must specify the **probe radius (mm), deformation (mm), force (N), and tissue thickness (mm)**. The calculator automatically solves for the **equilibrium modulus (MPa)** (light blue), which can then be copied into the tan colored **equilibrium contact modulus (MPa)** cell.

##### Step 3

The Solver function can now be run to optimize the biphasic model. Open the Solver. We want to minimize the error by changing the tensile modulus and the permeability values.

Set Objective: **\$K\$15**

To: **Min**

By Changing Variable Cells: **\$F\$15:\$G\$15**

Make Unconstrained Variables Non-Negative **No**

Select a Solving Method: **GRG Nonlinear**

#### **Step 4**

Assuming a successful iteration the template is complete. The **OUTPUTS**, light blue, are the **permeability (mm<sup>4</sup>/Ns), tensile modulus (MPa), modulus ratio, coefficient of determination R<sup>2</sup> and a plot of the data and the model fit.**

#### **Troubleshooting**

There is one main error that may occur in this tab.

- 1) Solver cannot converge
  - a. Check your objective and variable cells.
  - b. Change your initial guesses as they may be out of range: a higher tensile modulus = a higher asymptotic contact modulus and the permeability affects the inflection point. Note that it may be easier, visually, to manually manipulate the data with a logarithmically scaled X-axis.
  - c. Try checking the box to **Make Unconstrained Variables Non-Negative.**

#### **V. References**

- [1] Moore, A. C., and Burris, D. L., 2014, "Experimental characterization of biphasic materials using rate-controlled Hertzian indentation," Tribology International.
- [2] Stevanovic, M., Yovanovich, M. M., and Culham, J. R., 2001, "Modeling Contact between Rigid Sphere and Elastic Layer Bonded to Rigid Substrate," IEEE TRANSACTIONS ON COMPONENTS AND PACKAGING TECHNOLOGIES, 24, pp. 207-212.